Characterizing Magnetized Turbulence in Molecular Clouds (and Galaxies)

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Outline

• Dispersion of magnetic fields
  • Separation of turbulent and large-scale fields through structure functions
  • Example: the Chandrasekhar-Fermi technique

• Application/results
  • Single-dish - OMC-1, CSO/SHARP
    • Turbulence correlation length
    • Turbulent/ordered field energy ratio (CF equation)
  • Interferometry - SMA
    • Magnetized turbulent power spectrum
    • Ambipolar diffusion scale
  • Single-dish (Effelsberg) + Interferometry (VLA)
    • M51 - Anisotropic turbulence
Polarization Maps - what are they good for?

OMC-1 - SHARP, 350 and 450 µm

E field

B field

Structure Functions

- Common for studying turbulence
  - Nice properties for power-law power spectra with stationary signals
- Have been used in astrophysics for some time
  - Molecular clouds
  - Radio Astronomy
    - Beck et al. (1999) → Intensity maps (Stokes I, Q, and U)
Structure Functions

Given a polarization map

Angle $\Phi(r) \rightarrow \mathbf{B}$ (plane of the sky)

The Angular Structure Function (stationarity and isotropy)

$$\langle \Delta \Phi^2(\ell) \rangle = \frac{1}{N(\ell)} \sum_{N(\ell) \text{ pairs}} \left[ \Phi(r) - \Phi(r + \ell) \right]^2$$

If $\mathbf{B} = \mathbf{B}_t + \mathbf{B}_0$ (turbulent and ordered (large-scale) components)

$$\Rightarrow \langle \Delta \Phi^2(\ell) \rangle = \langle \Delta \Phi_t^2(\ell) \rangle + \langle \Delta \Phi_0^2(\ell) \rangle$$

with statistical independence.

$$\Rightarrow 1 - \langle \cos[\Delta \Phi(\ell)] \rangle \approx \frac{\langle \Delta \Phi^2(\ell) \rangle}{2}$$
Structure Functions - Large-scale

polynomial fit (even powers)
Structure Functions - Turb.+large-scale

This - that = beam-broadened autocorrelation

beam	turbulence
Example - Chandra-Fermi Equation

\[ B_0 \approx \sqrt{4\pi \rho \sigma (v)} \left[ \frac{\langle B_t^2 \rangle}{\langle B_0^2 \rangle} \right]^{-1/2} \]

(Chandrasekhar-Fermi 1953)

\( \rho \): mass density

\( \sigma (v) \): velocity dispersion (one-dimension)

But the angular dispersion \( \delta \Phi \) relative to the ordered field determined with polarization maps is

\[ \delta \Phi \approx \left[ \frac{\langle B_t^2 \rangle}{\langle B_0^2 \rangle} \right]^{1/2} \]

or is it really the case?
Problems with the CF method

1. The models for $B_0$ are imperfect and introduce more errors in the determination of $\delta \Phi$. This is solved with the structure function.

Moreover

2. Signal integration along the line of sight and across the telescope beam
   - $\langle B_t^2 \rangle$ is underestimated due to averaging process
   - $B_0$ is therefore overestimated
ordered + turbulent fields

$B = B_0 + B_t$

$1 - \langle \cos[\Delta \Phi(\ell)] \rangle \approx \frac{\langle \Delta \Phi^2(\ell) \rangle}{2}$

OMC-1 with SHARP at 350 $\mu$m

OMC-1 - SHARP/CSO, 350 and 450 $\mu$m

$\chi^2$ fit to Gaussian model:

$\delta, \frac{\langle B_t^2 \rangle}{\langle B_0^2 \rangle}$.

$(\delta \Phi)^2 \approx \frac{\langle B_t^2 \rangle}{\langle B_0^2 \rangle}$

[Image of graphs showing $1 - \langle \cos(\phi) \rangle$ as a function of distance]
OMC-1 / SHARP - Results

\[ \delta \approx 7.3'' = 16 \text{ mpc} \quad \text{turbulent correlation length} \]

\[ N = \frac{\left( \delta^2 + 2W^2 \right) \Delta'}{\sqrt{2\pi} \delta^3} \approx 21 \quad \text{number of turbulent cells} \]

\[ \frac{\langle B_t^2 \rangle}{\langle B_0^2 \rangle} = \frac{1}{N} \frac{\langle B_t^2 \rangle}{\langle B_0^2 \rangle} \approx 0.013 \]

\[ \frac{\langle B_t^2 \rangle}{\langle B_0^2 \rangle} \approx 0.28 \quad \text{turbulent/ordered field energy ratio} \]

with Chandrasekhar-Fermi equation

\[ B_0 \approx \sqrt{4\pi \rho \sigma(v)} \left[ \frac{\langle B_t^2 \rangle}{\langle B_0^2 \rangle} \right]^{-1/2} \approx 760 \mu \text{G} \quad \text{plane of the sky} \]

with \( n = 10^5 \text{ cm}^{-3}, \ A = 2.3, \) and \( \sigma(v) = 1.85 \text{ km s}^{-1} \)
Turbulent Power Spectrum

\[ 1 - \langle \cos[\Delta \Phi(\ell)] \rangle \approx \frac{\langle \Delta \Phi^2(\ell) \rangle}{2} \]

but

\[ \Rightarrow \langle \cos[\Delta \Phi(\ell)] \rangle \equiv \frac{\langle \mathbf{B} \cdot \mathbf{B}(\ell) \rangle}{\langle \mathbf{B} \cdot \mathbf{B}(0) \rangle} \quad \Leftarrow \]

With a Fourier transform on the turbulent component

\[ \frac{\langle \mathbf{B} \cdot \mathbf{B}(\ell) \rangle}{\langle \mathbf{B}^2 \rangle} \quad \Rightarrow \quad \frac{1}{\langle \mathbf{B}^2 \rangle} \| H(k_v) \|^2 R_t(k_v) \left[ \equiv b^2(k_v) \right] \]

We can determine the turbulent power spectrum \( R_t(k_v) \) by deconvolution of the beam \( H(k_v) \)
Turbulent Power Spectrum - simulations

Fig. 2. Core column density for $\mu = 120$ (bottom panel), $\mu = 5$ (middle panel) and $\mu = 2$ (top panel) along the z-axis.

Fig. 3. Mean gas density within a sphere of radius $r$ as a function of $r$ for three different timesteps of the high resolution runs. The solid line is before the protostar formation while dotted and dashed lines correspond to later times. The straight line corresponds to the density of the singular isothermal sphere. The times are in Myr.

Hennebelle et al. 2011, A&A, 528, 72
Turbulent Power Spectrum - simulations

Structure Function

Power Spectrum

- dissipation
- linear
- autocorrelation
- log-log
- beam

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Turbulent Power Spectrum - NGC 1333/SMA

B-vectors
beam: 1.6″ x 1.0″
sampling: 0.2″

Houde et al. 2011

850 μm dust emission (SMA)

RA (J2000)

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dissipation?
Turbulent Power Spectrum - Orion KL/SMA

B-vectors

beam: 2.6” x 1.7”
sampling: 0.25”

dissipation?

Tang et al. 2010

Houde et al. 2011
Ambipolar Diffusion - Orion KL/SMA

$\delta_{AD} \approx 11$ mpc

beam
Magnetized Turbulence in Galaxies

M51 with Effelsberg (100m) + VLA

ordered + turbulent fields

\[ B = B_0 + B_t \]

Fletcher et al. 2011 (MNRAS)
M51 - Polarized Flux

Fletcher et al. 2011 (MNRAS)

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Houde et al. 2012

d = 7.6 Mpc
1" = 37 pc
\( \lambda = 6.2 \text{ cm} \)
4" beam
1" sampling
M51- Isotropic Turbulence

<table>
<thead>
<tr>
<th></th>
<th>Northeast</th>
<th>Centre</th>
<th>Southwest</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$ (pc)</td>
<td>...</td>
<td>$67 \pm 7$</td>
<td>$66 \pm 8$</td>
</tr>
<tr>
<td>$N$</td>
<td>...</td>
<td>$13 \pm 3$</td>
<td>$14 \pm 4$</td>
</tr>
<tr>
<td>$\overline{B}_t^2 / \overline{B}^2$</td>
<td>$0.028 \pm 0.002$</td>
<td>$0.088 \pm 0.026$</td>
<td>$0.072 \pm 0.025$</td>
</tr>
<tr>
<td>$B_t^2 / B_0^2$</td>
<td>...</td>
<td>$1.28 \pm 0.29$</td>
<td>$1.08 \pm 0.29$</td>
</tr>
<tr>
<td>$B_t / B_0$</td>
<td>...</td>
<td>$1.13 \pm 0.13$</td>
<td>$1.04 \pm 0.14$</td>
</tr>
</tbody>
</table>

From $\sigma_{RM}$ analysis, Fletcher et al. get

$\delta \approx 50$ pc and $\frac{B_t}{B_0} \approx 1.2 - 1.5$
Consider all three regions at once ➔ more vectors
M51 - Anisotropic Turbulence

Houde et al. 2012
M51- Anisotropic Turbulence

Cho, Lazarian, and Vishniac 2002

Houde et al. 2012

no beam

fit

Generated

Perpendicular Direction

Parallel Direction (grid units)
M51- Anisotropic Turbulence

\[ \delta_{\parallel} \approx 98 \pm 5 \text{ pc} \]
\[ \delta_{\perp} \approx 54 \pm 3 \text{ pc} \]
\[ \frac{\delta_{\parallel}}{\delta_{\perp}} \approx 1.87 \pm 0.14 \]
\[ N \approx 15 \pm 2 \]
\[ \frac{\overline{B}_t^2}{\overline{B}_0^2} \approx 0.06 \pm 0.01 \]
\[ \frac{B_t^2}{B_0^2} \approx 1.02 \pm 0.08 \]
\[ \frac{B_t}{B_0} \approx 1.01 \pm 0.04 \]
Summary

• Angular dispersion function allows the separation of the turbulent and ordered components of the magnetic field without assuming any model for the latter.

• We can also account for the signal integration process along the line of sight and across the telescope beam.

• With high-enough resolution data ⟷ determination of the magnetized turbulent power spectrum (e.g., correlation length, inertial range index, dissipation scale).

• But we need even higher resolution (ALMA) and “larger” single-dish observatories, as well as an increase in the number of “vectors” (SOFIA and CCAT) for anisotropy measurements.
Merci!