MHD Turbulence Theory and Selected Implications

Alex Lazarian (Astronomy, Physics and CMSO)

Special Thanks to G. Eyink, G. Kowal, E. Vishniac
ISM reveals Kolmogorov spectrum of density fluctuations.

**Density fluctuations**

**ISM Turbulence Spectrum**

Slope $\sim -5/3$

**Electron density spectrum**

WHAM emission: density fluctuations

Chepurnov & Lazarian 2009

**Modified from Armstrong, Rickett & Spangler (1995)**

**Scintillations and scattering**

Density fluctuations also extreme scattering and absorption events which can be related to small ionized and neutral structures: SINS
Turbulence is a chaotic order

Turbulence = \sum eddies
Flows get turbulent for large Reynolds numbers

\[ \text{Re} = \frac{LV}{\nu} = \frac{(L^2/\nu)}{(L/V)} = \frac{\tau_{\text{diff}}}{\tau_{\text{eddy}}} \]

Point for numerical simulations: flows are similar for similar Re. Numerical Re < $10^4$, while Re of astro flows > $10^{10}$
Astrophysical fluids are generically turbulent

\[ Re = \frac{LV}{\nu} = \frac{(L^2/\nu)}{(L/V)} = \frac{\tau_{\text{diff}}}{\tau_{\text{eddy}}} \]

Astrophysical flows have \( Re > 10^{10} \).
The studies of reconnection extrapolate from low resolution numerical simulations to very different astrophysical regimes.
Turbulence is both dynamically and scientifically important

“Turbulence is the last great unsolved problem of classical physics”

R. Feynman
Main Points

Basic Properties of MHD turbulence
Turbulence and Reconnection
Reconnection Diffusion and Star Formation
Main Points

Basic Properties of MHD Turbulence

Turbulence and Reconnection

Reconnection Diffusion and Star Formation
Main Points

Basic Properties of MHD turbulence
Turbulence and Reconnection
Reconnection Diffusion and Star Formation
Kolmogorov theory reveals order in chaos for incompressible hydro turbulence

\[ \left( \frac{V_l^2}{t_{cas,l}} \right) = \text{const} \]

\[ t_{cas,l} = \frac{l}{V_l} \]

\[ \left( \frac{V_l^3}{l} \right) = \text{const}, \quad V_l \sim l^{1/3} \]

Or, \( E(k) \sim k^{-5/3} \)

Re\(>>1\)

Viscosity is not important

Re\(\sim1\)

Viscous dissipation

Still not important
Strong MHD turbulence is characterized by a “critical balance”.

- Critical balance
  \[
  \frac{l_\perp}{b_\perp} = \frac{l_\parallel}{B_0}
  \]

- Constancy of energy cascade rate
  \[
  \frac{b_\perp^2}{t_{\text{cas}}} = \text{const}
  \]

Or, \( E(k) \sim k^{-5/3} \)

Goldreich-Sridhar model (1995)
Alfvenic eddies get more and more elongated with the decrease of the scale

Cho, AL & Vishniac 2003
Arguments related to the nature of strong Alfvénic turbulence have faded away recently with GS winning.

Additional effects related to dynamical alignment (Boldyrev 2005, 2006), polarization (Beresynak & AL 2006), non-locality (Gogoveridze 2007) could potentially change the nature of MHD turbulence.

The highest resonance had Boldyrev’s work that claimed $k^{-3/2}$ spectrum and $l_\parallel \sim l_\perp^{1/2}$.

However, MHD turbulence is “diffusively local” (Beresnyak & AL) and therefore a larger extend of the bottleneck is expected. Thus the $k^{-3/2}$ is an artifact of the bottleneck and not the real slope.
First order structure functions

Demonstrates $r^{1/3}$ scaling

Simulation

$1024^3$, $2048^3$, $4096^3$
Anisotropy in SF

GS95:

**Boldyrev**

\[ l_{||} \sim l_{\perp}^{1/2} \]  

*(incompatible with the measurement)*
Tested model of MHD turbulence demonstrates anisotropy for Alfvenic and slow modes and isotropy for fast modes.

Equal velocity correlation contour (Cho & Lazarian 02)

- Alfvenic modes: \( \sim k^{-5/3} \)
- Slow modes: \( \sim k^{-5/3} \)
- Fast modes: \( \sim k^{-3/2} \)
Dust polarization from Planck in blue, and velocity gradients, rotated by 90 degrees in red. Grey is HI GALFA intensity.
Main Points

Basic Properties of MHD turbulence

Turbulence and Reconnection

Reconnection Diffusion and Star Formation
LV99 model extends Sweet-Parker model for turbulent astrophysical plasmas and makes reconnection fast

Turbulent reconnection:
Outflow is determined by field wandering.

Key element:
\( L/\lambda_{||} \) reconnection simultaneous events

\[ V_{rec} = V_A \frac{\Delta}{L_x} \]

\( \lambda_{||} \)

B dissipates on a small scale \( \lambda_{||} \) determined by turbulence statistics.

AL & Vishniac (1999)
henceforth referred to as LV99
LV99 model extends Sweet-Parker model for turbulent astrophysical plasmas and makes reconnection fast.

Turbulent reconnection:
Outflow is determined by field wandering.

Key element:
$L/\lambda \parallel$ reconnection simultaneous events

Without turbulence:
molecular diffusion coefficient $D \sim 10^{-5}$ cm$^2$/sec
($\leftarrow$ It’s for small molecules in water.)

$\Rightarrow$ Mixing time $\sim (\text{size of the cup})^2/D \sim 10^7$ sec $\sim 0.3$ year!

AL & Vishniac (1999) henceforth referred to as LV99
Reconnection is Fast: speed does not depend on Ohmic resistivity!

Lazarian & Vishniac 1999 predicts no dependence on resistivity

Kowal et al. 2009
The reconnection rate increases with input power of turbulence.

Lazarian & Vishniac (1999) prediction is $V_{\text{rec}} \sim P_{\text{inj}}^{1/2}$

Results do not depend on the guide field.

Kowal et al. 2012
Simulations demonstrate the development of turbulence through Kelvin-Helmholtz instability.

\[ V_\Delta \approx (C_K r_A)^{3/4} V_A y \beta^{1/2} \]

Expected reconnection rate, \( C_k \) is Kolmogorov constant, \( r_A \) is magnetization.
Measurements of the growth of reconnection layer agree with the prediction

\[ V_\Delta \approx (C_K r_A)^{3/4} V_{Ay} \beta^{1/2} \]

\[ r_A \approx 1/30, \ V_\Delta \approx 0.01 \]

New measurements in Kowal et al. 2015 agree well with this

\[ \frac{d\Delta}{dt} \approx g\beta^{1/2} (C_K r_A)^{3/4} V_{Ay} \]

AL et al. 2015
Big Implication: LV99 means that magnetic field in **turbulent fluids** is not frozen in.

Instead of flux freezing condition one should consider flux diffusion by turbulent flow. *This has dramatic consequences for many areas of astrophysics including star formation!*

Violation of magnetic field frozen in condition in turbulent fluids proven in Eyink (2011). *The equivalence of this and LV99 approach was demonstrated in Eyink, Lazarian & Vishniac 2011.*
Reconnection diffusion is a key process for star formation.

Astrophysical Implications of Turbulent Reconnection: from cosmic rays to star formation

A. Lazarian
Department of Astronomy, University of Wisconsin, 475 N. Charter St., Madison, WI 53706
Reconnection can provide diffusion with the turbulent diffusion rates
In the presence of weak turbulence and gravity magnetic field, diffuses away from the core.

Gravitational potential:

\[
\Psi(R \leq R_{\text{max}}) = -\frac{A}{R + R_*}
\]

\[
\Psi(R > R_{\text{max}}) = -\frac{A}{R_{\text{max}} + R_*}
\]

Santos de Lima et al. 2010

Models starting in equilibrium simulate the evolution of subcritical clouds, while those starting in non-equilibrium reproduce some features of supercritical collapse.
Turbulent Reconnection solves “magnetic braking catastrophe”
Turbulence and fast reconnection are interconnected; astrophysical implications are numerous

MHD turbulence makes reconnection fast

Reconnection diffusion and star formation